

# INTERACTIVE FORCES

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This short article is devoted to the dynamics of controlled (and, therefore, open) systems. The internal forces, which appear only in the presence of external free controls and depend explicitly on them, are considered. Such *interactive forces* may be regarded as feedbacks generated in the system by the external free controls. In particular, one is able to interpret the interactive controls as couplings of free controls with the action of interactive forces; such dynamical approach to the interactive systems often simplifies their analysis and allows to perform their synthesis on a systematic level. The field theoretic counterparts of interactive forces, the fields of interactive forces, are also considered.

The original impetus to write this article was an intention to construct a bridge between the note [1] on the physical dynamics of information processes and the series [2] on the mathematics of interactive games. However, it was understood soon that the subject is more general and should be classified as dynamics of controlled (and, therefore, open) systems as a part of mathematical physics. Such approach enlarges the area of researches, makes the understanding deeper and physically pithy as well as allows to adopt simple mechanical models for analysis of interactive systems.

The new proposed concept of the dynamics of controlled system is one of the interactive forces. Interactive forces are internal forces in the controlled systems, which appear only in the presence of external free controls and explicitly depends on them. In classical mechanics with constraints one is able to treat the reactions of constraints as interactive forces if the external controls are realized as forces themselves. This example is simplest and other ones may be regarded as its sophisticated generalizations. One can look for interactive forces in mathematical psychology, mathematical economics, mathematical sociology and other behavioral sciences, where the nontrivial reactions of the systems on the external controls are considered. The dynamical approach to the interactive systems, the controlled systems, in which free controls are coupled with the unknown or incompletely known feedbacks, thus interpreted as an action of the interactive forces, symplifies their analysis and allows to perform their synthesis on a systematic level.

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1991 *Mathematics Subject Classification.* 70Q05 (Primary) 70G50, 90D25 (Secondary).

*Key words and phrases.* Dynamics, Controlled systems, Open systems, Interactive forces, Interactive systems, Fields of interactive forces.

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Of course, the mechanical approach to the interactive forces does not substitute the physical analysis of their nature, which may be rather different even for mechanically similar systems. Also such approach has its evident methodological boundaries when one has deal with interactive systems, which may be regarded as complex systems with incomplete information [2], and is often practically useful only together with other approaches. However, the explication of this new “coordinate axis” for the description of interactive systems should not be underestimated.

## 1. Interactive forces.

**Definition 1.** *Interactive forces* are internal forces in the controlled systems, which appear only in the presence of external free controls and explicitly depends on them.

*Example 1.* In classical mechanics with constraints one is able to treat the reactions of constraints as interactive forces if the external controls are realized as forces themselves.

*Example 2 (A quantum mechanical example).* In quantum mechanics of continuously observed particle one may treat the Belavkin counterterm in quantum Hamiltonian [3] as corresponding to the interactive forces.

*Remark 1.* One is able to consider the fields of interactive forces in the Faraday-Maxwell sense.

**2. Interactive systems.** The concept of an interactive control was recently proposed by the author [2] to take into account the complex composition of controls of a real human person, which are often complicated couplings of his/her cognitive and known controls with the unknown subconscious behavioral reactions. This formalism is applicable also to the description of external unknown influences and, thus, is useful for problems in computer science (e.g. the semi-artificially controlled distribution of resources), mathematical economics (e.g. the financial games with unknown dynamical factors) and sociology (e.g. the collective decision making).

**Definition 2 [2].** An *interactive system* (with  $n$  *interactive controls*) is a control system with  $n$  independent controls coupled with unknown or incompletely known feedbacks (the feedbacks as well as their couplings with controls are of a so complicated nature that their can not be described completely).

Below we shall consider only deterministic and differential interactive systems. In this case the general interactive system may be written in the form:

$$\dot{\varphi} = \Phi(\varphi, u_1, u_2, \dots, u_n),$$

where  $\varphi$  characterizes the state of the system and  $u_i$  are the interactive controls:

$$u_i(t) = u_i(u_i^\circ(t), [\varphi(\tau)]|_{\tau \leq t}),$$

i.e. the independent controls  $u_i^\circ(t)$  coupled with the feedbacks on  $[\varphi(\tau)]|_{\tau \leq t}$ . One may suppose that the feedbacks are integrodifferential on  $t$ .

However, it is reasonable to consider the *differential interactive systems*, whose feedbacks are purely differential. It means that

$$u_i(t) = u_i(u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t)).$$

A reduction of general interactive systems to the differential ones via the introducing of the so-called *intention fields* was described in [2]. Such intention fields may be considered as hidden parameters. They are often considered in the non-behaviourist psychology on the qualitative level. One is able to interpret them as internal (individual) as well as external (e.g. collective) parameters.

The interactive systems introduced above may be generalized in the following way, which leads to the *indeterminate interactive systems*, is based on the idea that the pure controls  $u_i^\circ(t)$  and the interactive controls  $u_i(t)$  should not be obligatory related in the considered way. More generally one should only postulate that there are some time-independent quantities  $F_\alpha(u_i(t), u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t))$  for the independent magnitudes  $u_i(t)$  and  $u_i^\circ(t)$ . Such claim is evidently weaker than one of Def.2. For instance, one may consider the inverse dependence of the pure and interactive controls:  $u_i^\circ(t) = u_i^\circ(u_i(t), \varphi(t), \dots, \varphi^{(k)}(t))$ .

The inverse dependence of the pure and interactive controls has a nice psychological interpretation. Instead of thinking of our action consisting of the conscious and unconscious parts and interpreting the least as unknown feedbacks which “dress” the first, one is able to consider our action as a single whole whereas the act of consciousness is in the extraction of a part which it declares as its “property”. So interpreted the function  $u_i^\circ(u_i, \varphi, \dots, \varphi^{(k)})$  realizes the “filtering” procedure.

**3. Interactive forces in interactive systems.** If the pure controls  $u_i^\circ(t)$  in an interactive systems have a dynamical character, i.e. are realized by the external forces, one is able to try to interpret the action of interactive controls  $u_i(t)$  as a sum of an action of pure controls and interactive forces  $F_\alpha(u_i^\circ(t), \varphi(t))$ . Precisely, let the system has the generalized Newtonian form

$$\begin{cases} \dot{\mathbf{q}} = \frac{1}{m}\mathbf{p} \\ \dot{\mathbf{p}} = F(\vec{u}, \mathbf{q}, \mathbf{p}) \end{cases}$$

where  $\mathbf{q}$  and  $\mathbf{p}$  are generalized coordinates and momenta, respectively, and  $\vec{u} = (u_1, \dots, u_n)$  is the vector of  $n$  independent interactive controls. Then this system may be represented as

$$\begin{cases} \dot{\mathbf{q}} = \frac{1}{m}\mathbf{p} \\ \dot{\mathbf{p}} = F(\vec{u}^\circ, \mathbf{q}, \mathbf{p}) + \sum_{\alpha} F_{\alpha}(\vec{u}^\circ, \mathbf{q}, \mathbf{p}) \end{cases}$$

where  $\vec{u}^\circ = (u_1^\circ, \dots, u_n^\circ)$  is the vector of  $n$  independent pure controls and  $F_\alpha$  are the interactive forces.

If the forces are potential one may consider dynamics in Hamiltonian form instead of the generalized Newtonian one. In this case the Hamiltonian  $H(\vec{u}, \mathbf{p}, \mathbf{q})$  is decomposed into the sum  $H(\vec{u}^\circ, \mathbf{p}, \mathbf{q}) + \sum_{\alpha} H_{\alpha}(u^\circ, \mathbf{p}, \mathbf{q})$ , where  $H_\alpha$  are the terms corresponding to the interactive forces.

This dynamical interpretation of interactive controls often simplifies an analysis of the interactive systems and allows to perform their synthesis on a systematic

level. For instance, one is able to apply various mechanical principles such as superposition principle or the third law of dynamics to the interactive systems. Moreover, and it should be specially emphasized that *the synthesis of interactive systems is straightforward if their dynamical nature is explicated*. This is just the same as for the classical mechanics, where the Hamiltonians for two isolated systems do not provide a sufficient information to reconstruct the Hamiltonian of interaction, whereas the knowledge of dynamical nature of both systems completely determines their interaction.

*Remark 2 (The actions of interactive forces).* An action of the pure control may be zero in the mechanical sense (i.e. it does not change the energy of system) or, otherwords, the pure control “does not work” (for instance, the corresponding force is magnetic-type of gyroscopic), whereas the interactive forces are acting and, therefore, change the energy of system. In this situation the whole action is simultaneously effective and energy-expenseless for the controller so one may say that the pure control is resulted only in such non-energetical *effort* whereas the interactive forces really *act*.

*Remark 3.* Note that the concept of a field of interactive forces is close to the concept of an intention field. However, one is able to consider the intention fields, which are not defined by fields of interactive forces. And there are fields of interactive forces (see examples 1,2 above), which are not constructed as intention fields in any interactive systems.

*Remark 4.* One may treat intention fields and fields of interactive forces in physical interactive information systems as certain “objectivizations” of *māyā*, the philosophical concept adapted in [1] for a description of dynamics of physical interactive information systems. So the intention fields and fields of interactive forces are mathematical and dynamical counterparts for each other in this particular case.

It is rather interesting to explicate the dynamical aspects of the perception processes and image understanding, which interactive game theoretic description was done in [4]. Indeed, any interior image in the subjective perceptual space of observer is a configuration of fields of interactive forces due to its self-generating character. Moreover, any exterior object also should be described in terms of such fields in view of its dynamical property to generate the interior images. This is certainly reminiscent of the old Ernst Mach’s ideas.

As it was mentioned in the preceeding articles the visualization of intention fields is an effective tool for the organization of interactive processes. The visualization of such fields, which correspond to the fields of interactive forces, explicates the dynamical aspect of this procedure. This allows to use the dynamical nature of visual perception as a tool for the visualization of interactive processes of another nature. On the other hand, an application of visualization to the visual perception itself may be regarded as a *dynamical exteriorization* of internal visual perception processes and, thus, is effective for their description and the understanding of their dynamical nature.

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